LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SECOND SEMESTER – **APRIL 2025**

PMT2MC03 – PARTIAL DIFFERENTIAL EQUATIONS

Date: 29-04-2025 Dept. No. Max. : 100 Marks
Time: 01:00 PM - 04:00 PM

	SECTION A – K1 (CO1)
	Answer ALL the questions **(5 x 1 = 5)**
1.	**Answer the following**
a)	State the auxiliary equation of Lagrange's partial differential equations.
b)	Describe the interior Dirichlet problem for a circle.
c)	State two occurrences of the diffusion equation.
d)	Define wave function.
e)	Write the Green's function for Laplace equation.
	SECTION A – K2 (CO1)
	Answer ALL the questions **(5 x 1 = 5)**
2.	**MCQ**
a)	The particular solution of $(D^2 - DD')\,u = \cos(x + 2y)$ is (i) $-\cos(x + 2y)$ (ii) $\cos(x + 2y)$ (iii) $\frac{-1}{2}\cos(x + 2y)$ (iv) $\frac{1}{2}\cos(x + 2y)$
b)	The Poisson equation is (i) $\nabla^2 V = -4G\rho$ (ii) $\nabla^2 V = -4\pi G\rho$ (iii) $\nabla^2 V = 4G\rho$ (iv) $\nabla^2 V = 4\pi G\rho$
c)	A condition where the flux of heat is prescribed on the boundary surface is called a (i) Rodrigue's condition (ii) Robin's condition (iii) Neumann's condition (iv) Bessel's condition
d)	Which method is commonly used to solve the wave equation analytically? (i) Laplace transforms (ii) Separation of variables (iii) Newton's method (iv) Runge-Kutta method
e)	The Helmholtz equation is a type of which partial differential equation (PDE)? (i) Parabolic PDE (ii) Hyperbolic PDE (iii) Elliptic PDE (iv) First-order PDE
	SECTION B – K3 (CO2)
	Answer any THREE of the following **(3 x 10 = 30)**
3.	Using the Lagrange method, find the solution of $y^2p - xyq = x(z - 2y)$.
4.	Solve the equation $(D^2 + 2DD' + D'^2 - 2D - 2D')u = \sin(x + 2y)$.
5.	Apply the separation of variables method to solve the Laplace equation in polar coordinates.
6.	Derive the D'Alembert's solution of one-dimensional wave equation.
7.	Using Green's theorem, prove that $G(\boldsymbol{r}_1, \boldsymbol{r}_2) = G(\boldsymbol{r}_2, \boldsymbol{r}_1)$.
	SECTION C – K4 (CO3)
	Answer any TWO of the following **(2 x 12.5 = 25)**
8.	Determine the necessary and sufficient condition for the existence of compatible systems of first order equations.
9.	Describe the Dirichlet problem for a rectangle and solve it.

10.	Solve the one-dimensional diffusion equation $\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}, -\infty < x < \infty, t > 0$ with an initial condition $T(x,0) = f(x)$.
11.	Explain the eigenfunction method for determining the Green's function to the boundary value problem $\nabla^2 u = f$ valid in certain region R subject to the boundary condition $u = g$ on ∂R where f and g are given functions.
	SECTION D – K5 (CO4)
	Answer any ONE of the following **(1 x 15 = 15)**
12.	Reduce the equation $(1+x^2)u_{xx} + (1+y^2)u_{yy} + xu_x + yu_y = 0$ to a canonical form.
13.	The ends *A* and *B* of a rod, 10 cm length, which are kept at temperatures $0^\circ\mathrm{C}$ and $100^\circ\mathrm{C}$ until the steady state condition prevails. Suddenly the temperature at the end *A* is increased to $20^\circ\mathrm{C}$ and the end *B* is decreased to $60^\circ\mathrm{C}$. Determine the temperature distribution in the rod at time *t*.
	SECTION E – K6 (CO5)
	Answer any ONE of the following **(1 x 20 = 20)**
14.	Consider the Laplace equation $\nabla^2 u = 0$. Formulate the function $u(r,\theta)$ subject to the conditions $u(a,\theta) = f(\theta)$ and *u* must be bounded as $r \to \infty$.
15.	Formulate the vibration of a string as a mathematical problem and solve it.

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